

Exponential Potentials and Attractor Solution of Dilatonic Cosmology

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Abstract We present the scalar-tensor gravitational theory with an exponential potential in which Pauli metric is regarded as the physical space-time metric. We show that it is essentially equivalent to coupled quintessence (CQ) model. However for baryotropic fluid being radiation there are in fact no coupling between dilatonic scalar field and radiation. We present the critical points for baryotropic fluid and investigate the properties of critical points when the baryotropic matter is specified to ordinary matter. It is possible for all the critical points to be attractors as long as the parameters λ and β satisfy certain conditions. To demonstrate the attractor behaviors of these critical points, We numerically plot the phase plane for each critical point. Finally with the bound on β from the observation and the fact that our universe is undergoing an accelerating expansion, we conclude that present accelerating expansion is not the eventual stage of universe. Moreover, we numerically describe the evolution of the density parameters Ω and the decelerating factor q , and computer the present values of some cosmological parameters, which are consistent with current observational data.

Keywords Exponential potential · Scalar-tensor · Dark energy · Critical point · Attractor

1 Introduction

The evidences for the existence of dark energy have been growing in past few years. Recently the WMAP three result [1] has dramatically shrunk the allowed volume in the parameter space. It shows that in a spatial flat universe the combination of WMAP and the

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Supernova legacy survey (SNLS) data yields a significant constraint on the equation of state of the dark energy $w = -0.97^{+0.07}_{-0.09}$. Though the Λ CDM model is still an excellent fit to the WMAP data, it still does not exclude other alternative model for the candidate of dark energy. Moreover the well-known fine-tuning and coincidence problems [2, 3] are yet unsolved in cosmological constant model. This motivates a wide range of theoretic studies to explain the observation: such as the conventional “quintessence” scalar field [4–8]; the k-essence field [9–16]; quintom model [17–23]; holograph dark energy [24–29]; Born–Infeld scalar or vector field theory [30–37]; phantom model [38–47] and so on. Additionally, some authors attempt to modify the conventional gravitational theory instead of involving the exotic matter [48–54].

In past several years the idea that dilaton field of the scalar-tensor gravitational theory as the dark energy has been proposed and discussed [55–63]. In our previous paper [64], we have considered a dilatonic dark energy model, based on Weyl-scaled induced gravitational theory. In that paper, we find that when the dilaton field is not gravitational clustered at small scales, the effort of dilaton can not change the evolutionary law of baryon density perturbation, and the density perturbation can grow from $z \sim 10^3$ to $z \sim 5$, which guarantees the structure formation. When dilaton energy is very small compared the matter energy, potential energy of dilaton field can be neglected. In this case, the solution of cosmological scale a has been found [65, 66]. In recent work [67], we consider the dilaton field with positive kinetic energy and with negative kinetic energy and find that the coupled term between matter and dilaton can’t affect the existence of attractor solutions. In this paper we will study the attractor properties of the dynamical system. The potential we choose for investigation is the exponential form for its important role in higher-order or higher-dimensional gravitational theories, string theories and Kaluza–Klein model (see the references in Ref. [68]). Though the possible cosmological roles of exponential potential have been investigated elsewhere [69–79], here we will investigate its cosmological implies in our dilatonic cosmology. With the constraint from the observation we conclude that the present accelerating expansion is not the eventual stage of universe.

2 Theoretical Model from Scalar-Tensor Gravitational Theory

The action of Jordan–Brans–Dicke theory is:

$$S = \int d^4x \sqrt{-\gamma} \left[\phi \tilde{R} - \omega \gamma^{\mu\nu} \frac{\partial_\mu \phi \partial_\nu \phi}{\phi} - \Lambda(\phi) - \tilde{L}_{\text{fluid}}(\psi) \right] \tag{1}$$

where \tilde{L}_{fluid} is the Lagrangian density of cosmic fluid, γ is the determinant of the Jordan metric tensor $\gamma_{\mu\nu}$, ω is the dimensionless coupling parameter, \tilde{R} is the contracted $\tilde{R}_{\mu\nu}$. The metric sign convention is $(-, +, +, +)$. The quantity $\Lambda(\phi)$ is a nontrivial potential of ϕ field. When $\Lambda(\phi) \neq 0$ the (1) describes the induced gravity. $\tilde{\rho}$ and \tilde{p} is respectively the density and pressure of cosmic fluid. However it is often useful to write the action in terms of the conformally related Einstein metric. We introduce the dilaton field σ and conformal transformation as follows:

$$\phi = \frac{1}{2\kappa^2} e^{\alpha\sigma}, \tag{2}$$

$$\gamma_{\mu\nu} = e^{-\alpha\sigma} g_{\mu\nu} \tag{3}$$

where $\alpha^2 = \frac{\kappa^2}{2\omega+3}$, $\kappa^2 = 8\pi G$. Using (2), (3), the action (1) becomes

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R(g_{\mu\nu}) + \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - W(\sigma) + L_{\text{fluid}}(\psi) \right] \tag{4}$$

where $L_{\text{fluid}}(\psi) = \frac{1}{2} g^{\mu\nu} e^{-\alpha\sigma} \partial_\mu \psi \partial_\nu \psi - e^{-2\alpha\sigma} V(\psi)$.

The transformation equations (2) and (3) are well defined for some ω as $-\frac{3}{2} < \omega < \infty$. The conventional Einstein gravity limit occurs as $\sigma \rightarrow 0$ for an arbitrary ω or $\omega \rightarrow \infty$ with an arbitrary σ .

The nontrivial potential of the σ field, $W(\sigma)$ can be a metric scale form of $\Lambda(\phi)$. Otherwise, one can start from (4), and define $W(\sigma)$ as an arbitrary nontrivial potential. $g_{\mu\nu}$ is the Pauli metric. Cho and Damour et al. pointed out that the Pauli metric can represent the massless spin-two graviton in scalar-tensor gravitational theory [80, 81]. Cho also pointed out that in the compactification of Kaluza–Klein theory, the physical metric must be identified as the Pauli metric because of the wrong sign of the kinetic energy term of the scalar field in the Jordan frame. The dilaton field appears in string theory naturally.

By varying the action equation (4), one can obtain the field equations of Weyl-scaled scalar-tensor gravitational theory.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{\kappa^2}{3} \left\{ \left[\partial_\mu \sigma \partial_\nu \sigma - \frac{1}{2} g_{\mu\nu} \partial_\rho \sigma \partial^\rho \sigma \right] - g_{\mu\nu} W(\sigma) + e^{-\alpha\sigma} \left[\partial_\mu \psi \partial_\nu \psi - \frac{1}{2} g_{\mu\nu} \partial_\rho \psi \partial^\rho \psi \right] - g_{\mu\nu} e^{-2\alpha\sigma} V(\psi) \right\}, \tag{5}$$

$$\square \sigma = \frac{dW(\sigma)}{d\sigma} - \frac{\alpha}{2} e^{-2\alpha\sigma} g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi - 2\alpha e^{-2\alpha\sigma} V(\psi), \tag{6}$$

$$\square \psi = -\alpha g_{\mu\nu} \partial_\mu \psi \partial_\nu \sigma + e^{-\alpha\sigma} \frac{dV(\psi)}{d\psi} \tag{7}$$

where “ \square ” denotes the D’Alembertian. We assume that the energy-momentum tensor $T_{\mu\nu}$ of cosmic fluid is

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu + p g_{\mu\nu} \tag{8}$$

where the density of energy

$$\rho = \frac{1}{2} \dot{\psi}^2 + e^{-\alpha\sigma} V(\psi) \tag{9}$$

the pressure

$$p = \frac{1}{2} \dot{\psi}^2 - e^{-\alpha\sigma} V(\psi). \tag{10}$$

In FRW metric $ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$, we can obtain following equations from (5–7):

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{\kappa^2}{3} \left(\frac{1}{2} \dot{\sigma}^2 + W(\sigma) + e^{-\alpha\sigma} \rho \right), \tag{11}$$

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{dW}{d\sigma} = \frac{1}{2} \alpha e^{-\alpha\sigma} (\rho - 3p), \tag{12}$$

$$\dot{\rho} + 3H(\rho + p) = \frac{1}{2}\alpha\dot{\sigma}(\rho + 3p), \tag{13}$$

$$\dot{H} = -\frac{\kappa^2}{2}[\dot{\sigma}^2 + e^{-\alpha\sigma}(\rho + 3p)] \tag{14}$$

where we specify the cosmic fluid ρ as the baryotropic matter ρ_b with a equation of state $p_b = w_b\rho_b$. $w_b = 0$ for ordinary matter, $w_b = \frac{1}{3}$ for radiation, $W(\sigma)$ is exponentially dependent on σ as $W(\sigma) = W_0e^{-\lambda\kappa\sigma}$. The effective energy density of dilaton scalar field is $\rho_\sigma = \frac{1}{2}\dot{\sigma}^2 + W(\sigma)$, the effective pressure of dilaton scalar field is $p_\sigma = \frac{1}{2}\dot{\sigma}^2 - W(\sigma)$ and $p_\sigma = w_\sigma\rho_\sigma$. Here we should note that in Weyl-scaled scalar-tensor gravitational theory the coupling of the baryotropic matter with dilaton field σ is more natural.

We can get the solution of (13) for the density of baryotropic energy:

$$\rho_b \propto e^{\frac{1}{2}\alpha(1+3w_b)\sigma} a^{-3(1+w_b)}. \tag{15}$$

For matter, $w_m = 0$, $\rho_m \propto e^{\frac{1}{2}\alpha\sigma} a^{-3}$, and for radiation, $w_r = \frac{1}{3}$, $\rho_r \propto e^{\alpha\sigma} a^{-4}$.

3 Critical Points and the Attractor Solution

In this section, we investigate the global structure of the dynamical system via a phase plane analysis.

We define

$$x = \frac{\kappa\dot{\sigma}}{\sqrt{6H}}, \quad y = \frac{\kappa\sqrt{W(\sigma)}}{\sqrt{3H}}, \quad z = \frac{\kappa\sqrt{e^{-\alpha\sigma}\rho_b}}{\sqrt{3H}}. \tag{16}$$

Then (12–14) can be written as a plane autonomous system:

$$\frac{dx}{dN} = \frac{\sqrt{6}\alpha}{4\kappa}(1 - 3w_b)z^2 - 3x + \frac{\sqrt{6}\lambda}{2}y^2 + 3x^3 + \frac{3}{2}(1 + w_b)xz^2, \tag{17}$$

$$\frac{dy}{dN} = -\frac{\sqrt{6}\lambda}{2}xy + 3x^2y + \frac{3}{2}(1 + w_b)yz^2, \tag{18}$$

$$\frac{dz}{dN} = -\frac{\sqrt{6}\alpha}{2\kappa}xz + \frac{\sqrt{6}\alpha}{4\kappa}(1 + 3w_b)xz - \frac{3}{2}(1 + w_b)z + 3x^2z + \frac{3}{2}(1 + w_b)z^3, \tag{19}$$

where $N = \ln(a)$, the constraint equation (11) becomes:

$$x^2 + y^2 + z^2 = 1 \tag{20}$$

the density parameter of σ field is

$$\Omega_\sigma = x^2 + y^2 \tag{21}$$

the equation of state of the dilaton field σ is:

$$w_\sigma = \frac{x^2 - y^2}{x^2 + y^2}. \tag{22}$$

In order to investigate the expansive behavior of scale factor a , we also represent the decelerating factor:

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{3}{2} \left[(1 - w_b)x^2 - (1 + w_b)y^2 + \left(w_b + \frac{1}{3} \right) \right]. \tag{23}$$

Using (20), we rewrite (17–19) as follows:

$$\begin{aligned} \frac{dx}{dN} = & -3x + \lambda\sqrt{\frac{3}{2}}y^2 + \frac{3}{2}x[2x^2 + (1 + w_b)(1 - x^2 - y^2)] \\ & + \frac{\sqrt{6}}{4}\beta(1 - 3w_b)(1 - x^2 - y^2), \end{aligned} \tag{24}$$

$$\frac{dy}{dN} = y \left(-\sqrt{\frac{3}{2}}\lambda x + \frac{3}{2}[2x^2 + (1 + w_b)(1 - x^2 - y^2)] \right) \tag{25}$$

where $\beta = \alpha/\kappa$. It is more convenient to investigate the global properties of the dynamical system (24), (25) than (17–19). Because the case $y < 0$ corresponds to contracting universes, we will only consider the case $y \geq 0$ in the following discussion.

We can generally find five fixed points(critical points) where dx/dN and dy/dN both equal 0 (Table 1).

Depending on the different values of w_b , λ and β , we will study the stability of these critical points. Before investigating the stability of critical points, we should point out that for radiation $w_b = \frac{1}{3}$, $\rho_r = e^{\alpha\sigma} a^{-4}$, there are no coupling between dilaton field σ and radiation in (11). Since $\rho_\sigma = 3p_\sigma$, the right of (12) equals zero and therefore the radiation density does not contribute to (12). In this case, the form of (11), (12), (24), (25) are very similar with conventional scalar field and the corresponding stability of the critical points are presented in [68]. Therefore we only pay attention to the case $w_b = 0$ (matter). In this case, we will find that the properties of critical points are more distinctive than the case $w_b = \frac{1}{3}$.

We present the critical points and its properties in following Table 2: where $A = \frac{\beta^2 - 2\lambda\beta + 12}{(2\lambda - \beta)^2}$, $B = -1 + \frac{12}{\beta^2 - 2\lambda\beta + 12}$ and $C = \frac{(2\lambda - \beta)(\lambda + \beta)}{(2\lambda - \beta)^2}$. We define $\lambda_{\pm} = \lambda \pm \sqrt{\lambda^2 - 6}$, $\beta_{\pm} = \frac{\beta \pm \sqrt{\beta^2 + 48}}{4}$.

From Table 2, we know that for every critical point, they are all possible to be stable, but for every value of the parameters λ and β , there is one and only one stable critical point; the others are unstable.

P1 is a stable point (Fig. 1), which corresponds to a late-time decelerating rolling attractor solution where neither the dilaton nor the baryotropic fluid entirely dominates the evolution. Note that $\Omega_\sigma = \frac{\beta^2}{6}$, which is irrelevant to the potential parameter λ .

The properties of P2 (Fig. 2) and P3 (Fig. 3) are very similar, they both correspond to a late-time decelerating attractor solutions. In this case the right of (11) is dominated by the

Table 1

Points	P1	P2	P3	P4	P5
x	$\frac{\sqrt{6}\beta(1-3w_b)}{6(1-w_b)}$	1	-1	$\frac{\sqrt{6}}{6}\lambda$	$\frac{\sqrt{6}(1+w_b)}{2\lambda-(1-3w_b)\beta}$
y	0	0	0	$(1 - \frac{\lambda^2}{6})^{1/2}$	$\frac{\sqrt{\beta^2(3w_b-1)^2+2\lambda\beta(3w_b-1)-6(w_b^2-1)}}{2\lambda-(1-3w_b)\beta}$

Table 2

Points	x	y	Stability	Ω_σ	ω_σ	q
P1	$\frac{\sqrt{6}}{6}\beta$	0	$\lambda_- < \beta < \sqrt{6}(\lambda > \sqrt{6})$ or $-\sqrt{6} < \beta < \lambda_+(\lambda < -\sqrt{6})$	$\frac{\beta^2}{6}$	1	$\frac{\beta^2+2}{4}$
P2	1	0	$\lambda > \sqrt{6}$ and $\beta > \sqrt{6}$	1	1	2
P3	-1	0	$\lambda < -\sqrt{6}$ and $\beta < -\sqrt{6}$	1	1	2
P4	$\frac{\sqrt{6}}{6}\lambda$	$(1 - \frac{\lambda^2}{6})^{1/2}$	$\lambda^2 < 6$ and $\beta^2 < 24$	1	$\frac{\lambda^2}{3} - 1$	$\frac{\lambda^2}{2} - 1$
P5	$\frac{\sqrt{6}}{6}\lambda$	$(1 - \frac{\lambda^2}{6})^{1/2}$	$\max(\beta_-, -\sqrt{6})$ $< \lambda < \min(\beta_+, \sqrt{6})$	1	$\frac{\lambda^2}{3} - 1$	$\frac{\lambda^2}{2} - 1$
P6	$\frac{\sqrt{6}}{2\lambda-\beta}$	$[\frac{\beta^2-2\lambda\beta+6}{(2\lambda-\beta)^2}]^{1/2}$	see (31)	A	B	C

Fig. 1 The phase plane for $\lambda = 3, \beta = 2$ (P1). The late-time attractor is rolling solution with $x_1 = \sqrt{6}/3, y_1 = 0$, where $\Omega_\sigma = 2/3, q = 3/2$ and $w_\sigma = 1$

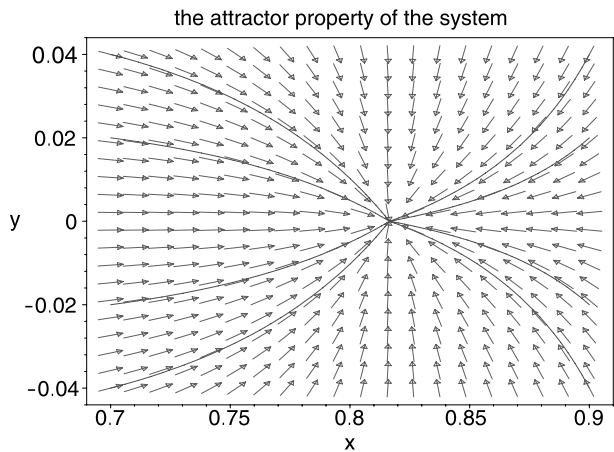
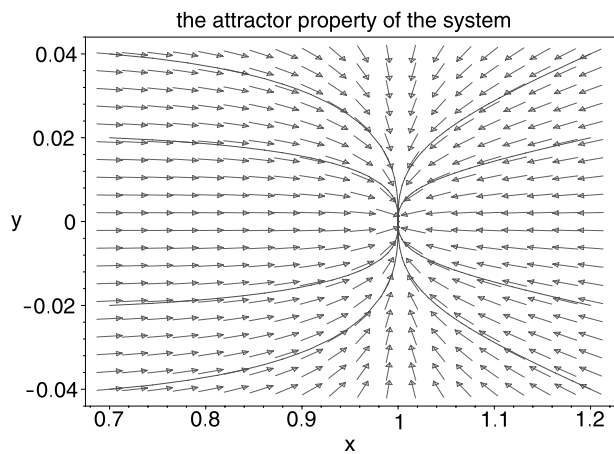


Fig. 2 The phase plane for $\lambda = 3, \beta = 4$ (P2). The late-time attractor is a dilaton field dominated solution with $x_2 = 1, y_2 = 0$, where $\Omega_\sigma = 1, q = 2$ and $w_\sigma = 1$



kinetic energy of the dilaton field, which are different with the critical point P1. All the three points describe the dilaton field behaving as a “stiff” matter with a equation of state $w_\sigma = 1$. These three points are obtained from $y = 0$ in (25).

Fig. 3 The phase plane for $\lambda = -5, \beta = -6$ (P3). The late-time attractor is a dilaton field dominated solution with $x_3 = -1, y_3 = 0$, where $\Omega_\sigma = 1, q = 2$ and $w_\sigma = 1$

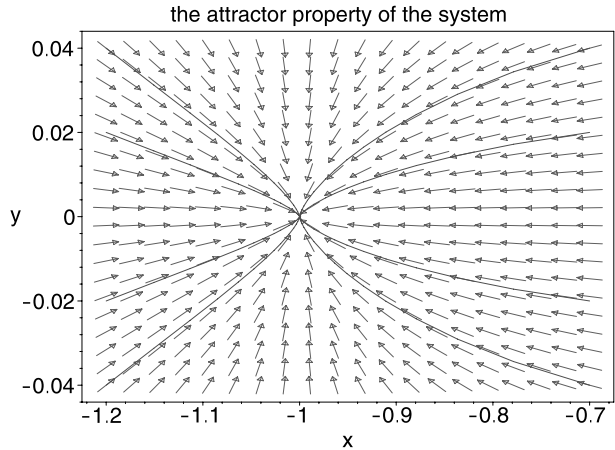
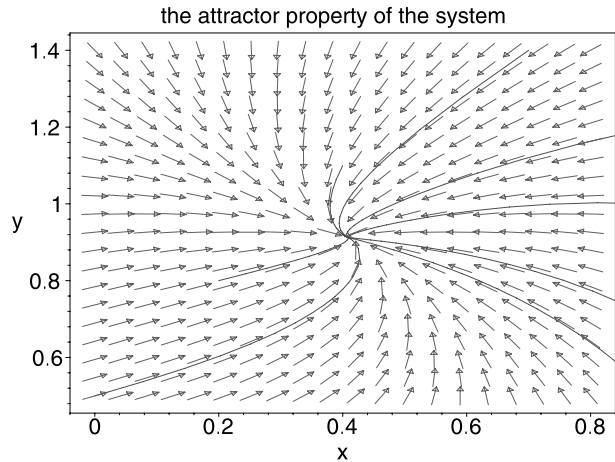


Fig. 4 The phase plane for $\beta = 2\lambda = 2$ (P4). The late-time attractor is a dilaton field dominated solution with $x_4 = \sqrt{1/6}, y_4 = \sqrt{5/6}$, where $\Omega_\sigma = 1, q = -1/2$ and $w_\sigma = -2/3$



However if

$$-\sqrt{\frac{3}{2}}\lambda x + \frac{3}{2}[2x^2 + (1 + w_b)(1 - x^2 - y^2)] = 0 \tag{26}$$

we can obtain other critical points. From (24), (25) ($dx/dN = 0, dy/dN = 0$) we get following equation:

$$(\beta - 2\lambda)x^2 - \frac{\sqrt{6}}{6}[\lambda(\beta - 2\lambda) - 6]x - \lambda = 0. \tag{27}$$

Solving (27), we can obtain critical points P4–P6. there are two different cases.

Case 1. If $\beta = 2\lambda$, there exists only one solution (P4, Fig. 4) for (27). It is a stable node point for $|\lambda| < \sqrt{6}$ and corresponds to an accelerating expansive universe for $|\lambda| < \sqrt{2}$.

Case 2. If $\beta \neq 2\lambda$, there are two solutions for (27) and therefore exist two critical points (P5, P6).

Fig. 5 The phase plane for $\lambda = 1/3, \beta = 8$ (P5). The late-time attractor is a dilaton field dominated solution with $x_5 = \sqrt{1/54}, y_5 = \sqrt{53/54}$, where $\Omega_\sigma = 1, q \approx -0.944$ and $w_\sigma = -0.963$

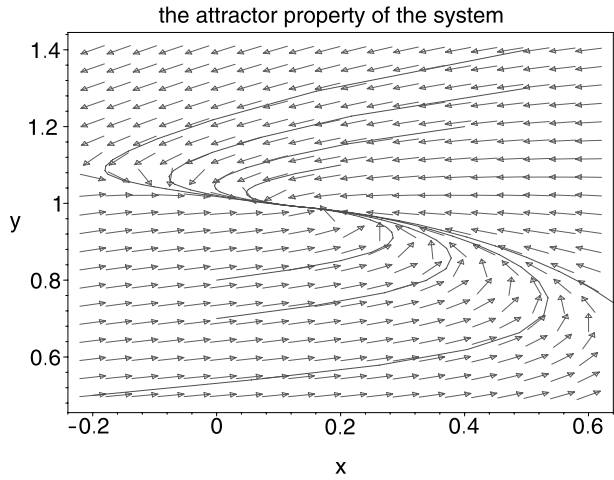
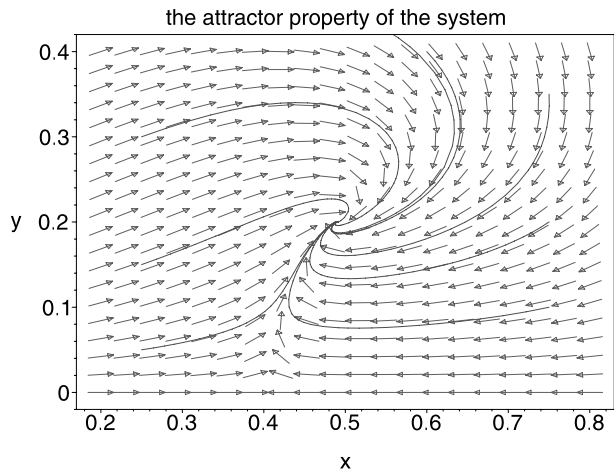


Fig. 6 The phase plane for $\lambda = 3, \beta = 1$ (P6, decelerating expansion). The late-time attractor is scaling solution with $x_6 = \sqrt{6}/5, y_6 = 1/5$, where $\Omega_\sigma = 7/25, q = 4/5$ and $w_\sigma = 5/7$



For critical point P5 it is stable for $\max(\beta_-, -\sqrt{6}) < \lambda < \min(\beta_+, \sqrt{6})$. P5 corresponds to a late-time attractor solution (Fig. 5) where the density of dilaton field will dominate the universe. The universe will undergo a stage of accelerating expansion if $|\lambda| < \sqrt{2}$.

There are two constraints on the parameters β and λ for point P6. Since $y_5 = [\frac{\beta^2 - 2\lambda\beta + 6}{(2\lambda - \beta)^2}]^{1/2}$, we have the first constraint:

$$\beta^2 - 2\lambda\beta + 6 \geq 0. \tag{28}$$

The second constraint is from that: $\Omega_b = 1 - \Omega_\sigma \geq 0$. So we have:

$$2\lambda^2 - \lambda\beta - 6 \geq 0. \tag{29}$$

To be a stable point for P6 (Fig. 6), the parameters λ and β have to satisfy another constraint (from the analysis of the stability of (24), (25)):

$$(2\lambda - \beta) \left[\lambda - \frac{1}{2}(\beta^2 - \beta^3)^{1/3} - \frac{1}{2}\beta \right] > 0. \tag{30}$$

From (28–30) we have following constraints on λ and β :

$$\begin{aligned} \lambda &\geq \beta_+ && \text{for } \beta < 0; \\ \lambda &\leq \beta_- && \text{for } \beta > 0; \\ \beta_- \geq \lambda &\geq \frac{3}{\beta} + \frac{\beta}{2} && \text{for } 0 > \beta \geq -\sqrt{6}; \\ \frac{3}{\beta} + \frac{\beta}{2} &\geq \lambda \geq \beta_+ && \text{for } \sqrt{6} \geq \beta > 0; \end{aligned} \tag{31}$$

Equation (31) is the condition for P5 being a stable point.

If the parameters λ and β also satisfy another relation:

$$(2\lambda - \beta)(\lambda + \beta) < 0 \tag{32}$$

the stable point will correspond to a late-time accelerating expansive universe where the dark energy can not dominate the universe entirely. From (31), (32), we get the range of λ and β for an accelerating expansive universe:

$$\begin{aligned} -\beta > \lambda \geq \beta_+ & \text{ for } \beta < -\sqrt{2}, \\ \beta_- \geq \lambda \geq -\beta & \text{ for } \beta > \sqrt{2}. \end{aligned} \tag{33}$$

However if $2\lambda^2 - \lambda\beta - 6 = 0$ ($\Delta = 0$ in (27)), the two different critical points (P5 and P6) become one same point. In this case we find that the largest eigenvalue for linear perturbations vanishes and we must consider higher-order perturbations about the critical point to determine its stability.

In order to clearly understand the attractor properties of these six critical points, we plot their phase planes when the values of λ and β lie in the corresponding range presented in Table 2.

4 Constraint from observation

Firstly we consider the observational data that the present solar-system gravitational experiments set (at the 1σ confidence level) a tight upper bound on $\beta_{\text{solar-system}}^2 < 10^{-3}$ [82–84]. So we have:

$$\beta^2 < 10^{-3}. \tag{34}$$

Therefore with the constraint on β and the attractor conditions presented in Table 2, we know that there only exists four possible late-time attractors (P1, P4, P5, P6), corresponding to four different cosmological destinies:

- A. The critical point P1. It is a stable node for $\lambda_- < \beta < \sqrt{6}(\lambda > \sqrt{6})$ or $-\sqrt{6} < \beta < \lambda_+(\lambda < -\sqrt{6})$. Considering the upper bound equation (34) on β , it requires $|\lambda| > 94.88$. In this case, the universe will be nearly entirely dominated by ordinary matter ($\Omega_\sigma = \frac{\beta^2}{6} < \frac{1}{6000}$) and end with a decelerating expansion. Therefore the present accelerating expansion is only a transient regime.
- B. The critical point P4. In this case $\beta = 2\lambda$, so from (34), we have $\lambda^2 < \frac{1}{4} \times 10^{-3}$. In this case the universe will be entirely dominated by dilatonic scalar field ($\Omega_\sigma = 1$) and it is undergoing an accelerating expansion with the value of $w_\sigma \approx -1$ and $q \approx -1$. Since the current density parameter of dark energy $\Omega_{\text{DE}} \approx 0.7$, the current cosmological stage is not a final evolutive stage yet.

Fig. 7 The evolution of density parameter Ω with respect to N . $\lambda = 0.01$, $\beta = -0.02$. Solid line is for the dilaton field Ω_σ , dotted line is for radiation Ω_r and dashed line is for matter Ω_m

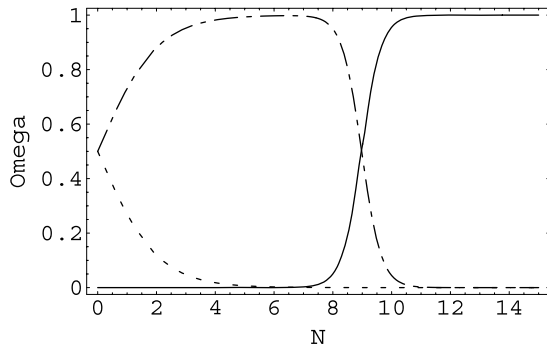
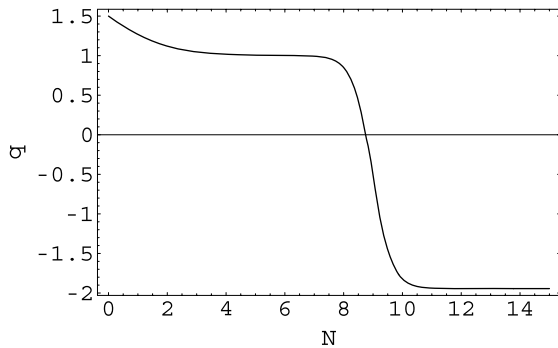


Fig. 8 The evolution of decelerating factor q with respect to N . It clearly shows that the universe underwent a decelerated regime at the recent past with the transition redshift $Z_T \simeq 0.670$



- C. The critical point P5. It is stable for $\max(\beta_-, -\sqrt{6}) < \lambda < \min(\beta_+, \sqrt{6})$. From (34), the constraint for λ is $1.73 > \lambda > -1.73$. In this case the universe will be entirely dominated by dilatonic scalar field ($\Omega_\sigma = 1$). Whether the universe finally undergoes an accelerating expansion is determined by the value of λ ($q = \frac{1}{2}(\lambda^2 - 2)$). Clearly the current cosmological stage is not a final evolutive stage yet.
- D. The critical point P6. From (33) we know that there is no accelerating expansion if β satisfies (34), so the current stage is still not a final evolution in this case.

The second constraint is from the fact that our universe is now undergoing an accelerating expansion and is dominated by the dark energy. The contribution of dark energy is $\Omega_\sigma \approx 0.7$ and its state parameter (ω_σ) is very close to -1 . So, we can exclude the points P1 and P6, which do not admit accelerating expansion. Therefore there only exist two critical points P4 and P5 which satisfy above constraints. So we can conclude that the universe will finally be completely dominated by dilatonic scalar field, hence the current cosmological state is not our ultimate destiny and we are all on the way toward final late-time attractor. Furthermore, We plot the evolution of density parameter Ω_σ , Ω_m and Ω_r (Fig. 7, where we consider the baryotropic matter ρ_b as matter and radiation, not only one). Figure 8 is the evolution of the decelerating factor q with respect to N , which quantitatively indicates that the universe is speeding up at present but underwent a decelerated regime at the recent past. Since $T_{eq} \simeq 5.64(\Omega_0 h^2) \text{ eV} \simeq 2.843 \times 10^4 \text{ K}$ (where we set $h = 0.71$), $T_0 \simeq 2.7 \text{ K}$, $a_i = a_{eq} = 1$, the scale factor at the present epoch a_0 would be nearly 1.053×10^4 , then we know at present epoch $N_0 = \ln a_0 \simeq 9.262$. We calculate the present value of $\Omega_{\phi_0} \simeq 0.701$, decelerating factor $q_0 \simeq -1.085$ and the transition redshift $Z_T \simeq 0.670$ in our model. It is consistent with current observation data.

5 Summary

In this paper, we studied the global properties of dilaton universe in the presence of dilatonic scalar field σ coupling to a baryotropic perfect fluid. We find the general solution (15) for the density of baryotropic energy. We show that if the baryotropic matter is radiation ($w_b = 1/3$) the stability of critical points are the same as the scalar field [68] which is no coupling to baryotropic matter. In this case there are in fact no coupling between σ field and radiation. However, if the baryotropic fluid is matter ($w_b = 0$), the situation is quite different. Since there exists coupling between σ field and matter, which is mathematically equivalent to the coupled quintessence model. It is possible for all the critical points to be stable but for every value of the parameters λ and β , there is one and only one critical point to be stable; the others are unstable. We plot the phase plane for each critical point and the results clearly show the attractor properties of these critical points. With the constraints from the observation data, we conclude that our present universe does not reach the ultimate cosmological state yet and we are all on the way toward final late-time attractor. Finally we present a numerical computations on the density parameter Ω and the decelerating factor q . All the results are consistent with current observation data.

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